Quantifiers and Predicates

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February 4, 2015

Try and express the following propositions using only the notions of addition, multiplication, and quantifier logic (with the universe taken to be $\mathbb{N}_0$). The following examples are all taken from Godel, Escher, Bach by Douglas Hofstadter, chapter VIII.

- 5 is prime.
- 2 is not a perfect square.
- 1729 is a sum of two cubes.
- No sum of two positive cubes is itself a cube.
- There are infinitely many prime numbers.
- 6 is even.

The next few work in the opposite direction: translate these to English, first by directly thinking about the quantifiers, then by doing some more interpretation. Next judge whether each statement is true or false.

- $\forall c : \exists b : (2 \cdot b = c)$
- $\forall c : \neg\exists b : (2 \cdot b = c)$
- $\forall c : \exists b : \neg\neg\neg(2 \cdot b = c)$
- $\neg\exists b : \forall c : (2 \cdot b = c)$
- $\exists b : \neg\forall c : (2 \cdot b = c)$
- $\exists b : \forall c : (2 \cdot b = c)$

A little later in the chapter Hofstadter gives some more translation problems. Again, you are only supposed to use addition, multiplication, and quantifiers in your answers.

- All natural numbers are equal to 4.
- There is no natural number which equals its own square.

- Different natural numbers have different successors.
- If 1 equals 0, then every number is odd.
- b is a power of 2.
- b is a power of 10.

Note that the last two of the above are quite difficult.