Verifying Robustness of Programs Under Structural Perturbations

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Motivation

- An attempt to synthesize the max function using PBE:
  - $(13, 15) \mapsto 15$
  - $(-23, 19) \mapsto 19$
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- Synthesized program: `P(a,b):=return b`
- Neither synthesized program, nor synthesizer are *robust*

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  would synthesize very different program
- Synthesize a robust program or develop robust synthesizer

Robustness Properties

- **Continuity**: small change to input $\Rightarrow$ small change to output

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Sort([1,4,3,6])=[1,3,4,6]
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- **Simultaneous Permutation**: permuting all inputs leaves output invariant (`Grade(responses,answers)`)

```
Grade([sqrt(x^2), 1/e, 6.5], [abs(x), e^-1, 13/2])=1
```

rearrange problem parts

```
Grade([1/e, 6.5, sqrt(x^2)], [e^-1, 13/2, abs(x)])=1
```

Verifying Continuity [1]

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if x ≥ 0 then
    r := y
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- Proof rule:

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- Only applicable to numerical perturbations

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- Requires specifying property in first-order logic
- Not optimized for 2-safety properties

Our Contributions

Goals:

- Reason about invariance under discrete perturbations
- Want to optimize for our specific problem

Results:

- Small sets of perturbations that "generate" all perturbations
  - Lists, binary search trees
- Formulate "invariance with respect to a function"
  - General, sound procedure
- Sanity checks and bug finding

Lists – Invariance under order

Given an array a

- Let a_{swap} be a with its first and second entry swapped
  - $[a[1], a[0], a[2], a[3], \ldots, a[n]]$
- Let a_{rot} be a rotated by 1
  - $[a[1], a[2], a[3], \ldots a[n], a[0]]$

Lemma: If for any a, $P(a) = P(a_{swap}) = P(a_{rot})$, then for any permutation a' of a, we have $P(a) = P(a')$.
Proof: Math [3]

Programs – Invariance under order

- maxList([x]) = x
- maxList([x, ...xs...]) = max(x, maxList(xs))

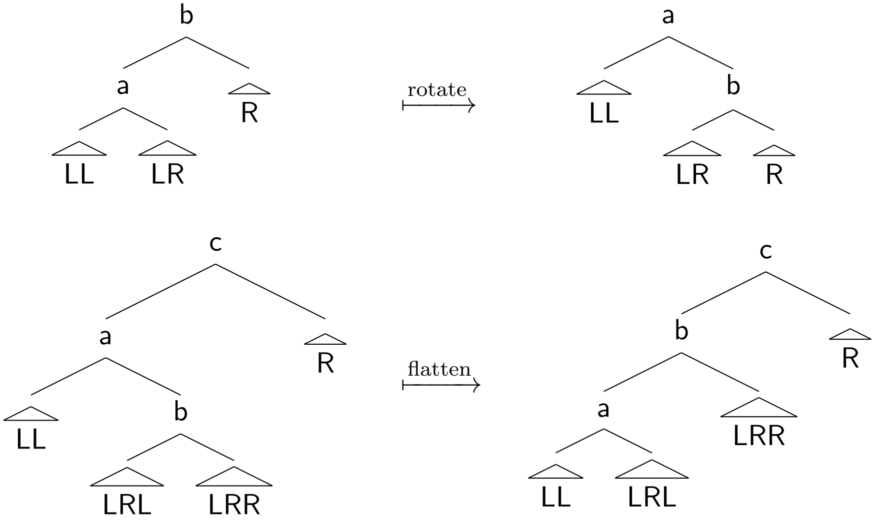
- Verifying maxList(a) = maxList(a_{swap}) has one case:

$$\begin{array}{rcl} maxList([x, y, ...xs...]) & \stackrel{?}{=} & maxList([y, x, ...xs...]) \\ || & & || \\ max(x, maxList([y, ...xs...])) & & max(y, maxList([x, ...xs...])) \\ || & & || \\ max(x, max(y, maxList(xs))) & & max(y, max(x, maxList(xs))) \\ || & & || \\ max(x, max(y, z)) & & max(y, max(x, z)) \end{array}$$

Binary Search Trees

- For lists, two simple permutations generated all permutations
- Goal: similar permutations for BSTs

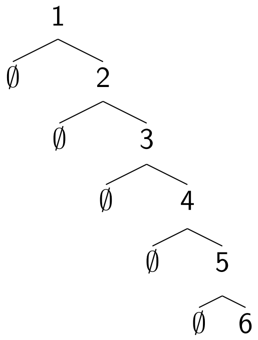
Binary Search Trees



Binary Search Trees

It suffices to show

- Every tree can be transformed into a “normal form” (i.e. list)
  - “flatten” straightens out the tree
  - “rotate” lets you straighten all the parts
- Every operation is reversable



Lists and Binary Search Trees

- Can check robustness under ALL permutations by checking just TWO permutations

More General Procedure

- Sets of permutations are case-by-case
- Goal: formulation of invariance
    - Useful
    - Easy to code/express
    - Checkable

More General Procedure

Invariance of a program $P : T \to Z$ relative to a function $f : T \to T'$

- $f(t)$ gives a "canonical representative" of t
- For concreteness, $f = \mathit{list} : \mathit{BST} \to \mathit{List}$

Observation: The following are equivalent:

- $\mathit{list}(x) = \mathit{list}(y) \implies P(x) = P(y)$
- There exists a program $\widetilde{P} : \mathit{Lists} \to Z$ such that $P(t) = \widetilde{P}(\mathit{list}(t))$

$$\begin{tikzcd} \mathit{BSTs} \arrow[rr, "P"] \arrow[dr, "\mathit{list}"] & & Z \\ & \mathit{Lists} \arrow[ur, dashed, "\widetilde{P}"] & \end{tikzcd}$$

More General Procedure

- Idea: Synthesize a witness to the invariance
  - A function $\widetilde{P} : \mathit{Lists} \to Z$
- P and *list* provide a *full specification* of $\widetilde{P}$
- Counterexample guided inductive synthesis [4]

$$\begin{tikzcd}
\mathit{BSTs} \arrow[rr, "P"] \arrow[dr, "\mathit{list}"'] & & Z \\
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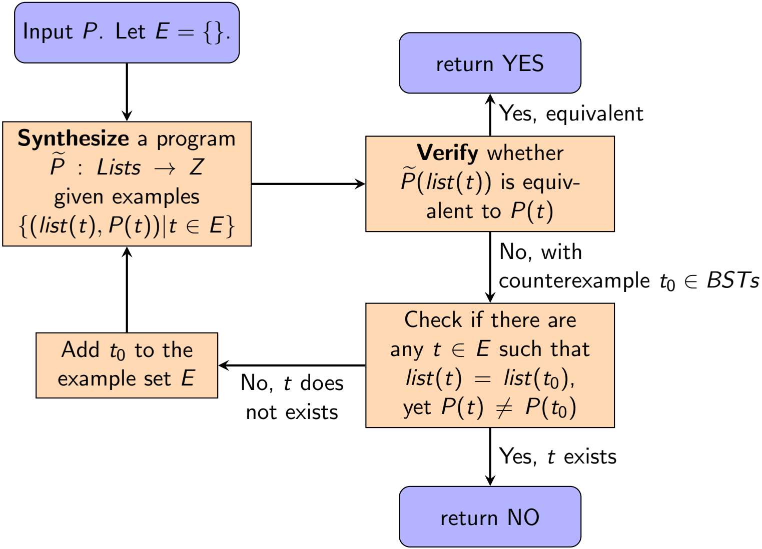

Future Directions

- Develop proof rules for discrete perturbations
- Improved handling of branching programs by Cartesian Hoare Logic
- Working implementation of Cartesian Hoare Logic
- Find more data structures with small perturbation sets
- Speed up our general procedure
- Synthesis for verification?
- Implement!

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