Exponential Communication Separations between Notions of Selfishness

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Setting — Algorithmic Mechanism Design

- Goal: Find a low-communication protocol to compute an outcome $a = f(t_1, \ldots, t_n)$, where each t_i known only to "player i"
- Twist: Player i must be *incentivized* to follow the protocol
  - t_i is player i's *type*
  - Players only want to maximize their *utility* $u_i(t_i, a)$ for the outcome a selected
- A *mechanism* consists of a protocol M & strategies $S_1(t_1), .., S_n(t_n)$
  - M computes $f(t_1, \ldots, t_n)$ when each player plays $S_i(t_i)$
  - We think of M as a game played among the n players
  - $S_i(t_i)$ denotes the strategy type t_i is meant to follow ("truth telling" strategy)
- There are two canonical definitions of "incentivization"
- A mechanism is **ex-post incentive compatible** (EPIC) if:

$$\forall t_i, s_i', t_{-i}: \qquad u_i\big(t_i,\ M(S_i(t_i), S_{-i}(t_{-i}))\big) \ge u_i\big(t_i,\ M(s_i', S_{-i}(t_{-i}))\big)$$

  - You want to tell the truth, *as long as the other players are following the protocol*
- A protocol is **dominant strategy incentive compatible** (DSIC) if:

$$\forall t_i, s_i', \mathbf{s_{-i}}: \qquad u_i\big(t_i,\ M(S_i(t_i), \mathbf{s_{-i}})\big) \ge u_i\big(t_i,\ M(s_i', \mathbf{s_{-i}})\big)$$

  - You want to tell the truth, *regardless of what "crazy strategies" other players use*

EPIC and DSIC: A simple example

- Function: two player, single item, second price auction
  - Environment: 2 players and a single indivisible item
  - Types: each player i values the item at $t_i \ge 0$
  - Utilities: if you win the item and pay price p, get utility $t_i - p$ (otherwise 0)
  - Function: the item should go to the player i with highest value t_i, charged a price equal to the value t_{3-i} of the other player
- **Protocol 1: Sequential auction.** Player 1 publicly announces their type t_1. Next, player 2 observes t_1 and then announces t_2.
  - If player 2 will bid t_2, it's in player 1's best interest to bid t_1 (so this is EPIC)
  - Suppose player 2 uses the following strategy:
    If player 1 bids 10, they bid 9. Otherwise, they bid 0.
  - If player 1's true value is 10, they'd rather bid 5 (so the mechanism is *not* DSIC).
- **Protocol 2: Sealed bid auction.** Agents submit bids simultaneously and privately
  - Each player's strategy corresponds to a single numerical bid, so truth telling is best for each player *regardless* of the strategies played (DSIC)
- Remark: Unlike protocols for non-strategic agents, have to keep track of *what agents know*, and consider when agents "break the protocol"

Goal: separate CC^{EPIC} and CC^{DSIC}

- Let $CC^{EPIC}(f)$ (resp. $CC^{DSIC}(f)$) denote the minimum communication cost of an EPIC (resp. DSIC) mechanism for f
- Trivially, we have $CC^{EPIC}(f) \le CC^{DSIC}(f)$, but little else known
  - [Fadel, Segal 09]: $CC^{DSIC}(f) \le \exp(CC^{EPIC}(f))$.
    Reason: For *simultaneous* protocols, EPIC and DSIC are equivalent
  - [Dobzinski 16]: In *certain* environments, $CC^{DSIC}(f) = \mathrm{poly}(CC^{EPIC}(f))$

Theorem:
There exists a function F such that $CC^{DSIC}(F) = \exp(CC^{EPIC}(F))$.

- Insight: The solution concept used can matter tremendously, constituting a "strategic resource" needed to compute f
- Extrapolation: DSIC may be too strong to hope for in many settings requiring interactive mechanisms

Building up to our construction F

- Goal: find an f with $CC^{EPIC} = \mathrm{poly}(m)$, but $CC^{DSIC} = \exp(m)$
- Idea: Modify **Protocol 1** so there's a *reason* to be sequential
- First attempt: environment where Alice wants a single set $S \subseteq [m]$ with $|S| = m/2$. Bob gives Alice S or $\overline{S}$, whichever Bob wants less.
  - Essentially an *indexing* problem
  - Mechanism M_1: Tell Bob which set S Alice wants. Bob responds with a single bit to determine if she gets S or $\overline{S}$.
  - M_1 is EPIC and low communication
  - M_1 isn't DSIC: Bob can give Alice $\overline{S}$ when she says S, and S when she says $\overline{S}$
- However, this function *does* have an efficient DSIC mechanism
  - Mechanism M_2: Alice publicly announces $\{S, \overline{S}\}$ (without saying which is which)
  - M_2 is DSIC: Alice get 0 utility from sets other than $\{S, \overline{S}\}$;
    Bob cannot condition on the difference between S and $\overline{S}$.
- Separating function F: *two disjoint copies* of the first attempt
  - Alice wants two sets $S_1 \subseteq M_1, S_2 \subseteq M_2$, with additive value across the two
  - For both $i = 1, 2$, Bob will give Alice S_i or $\overline{S_i}$ depending on his type
- Running two copies of M_2 gives an efficient EPIC mechanism for F
  - Exercise: this mechanism is *not* DSIC
  - **Theorem:** indeed *no* communication efficient mechanism for F is DSIC

An incorrect proof

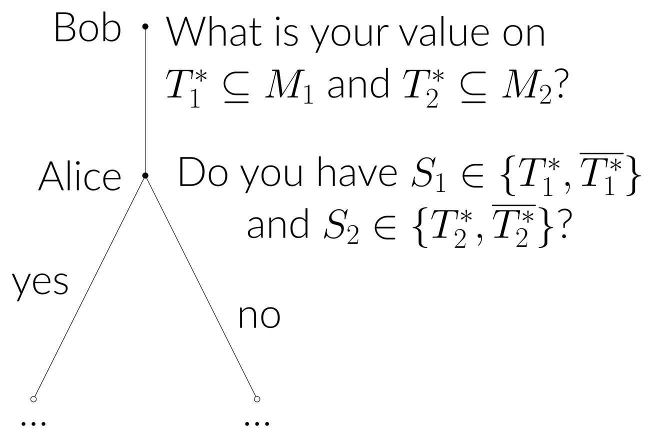
Claim: Any polynomial CC mechanism for F cannot be DSIC
Buggy proof: Any poly communication mechanism must allow Bob to respond to information provided by Alice. Consider the first point at which Alice conveys some information to Bob. Say that when Alice wants sets S_1, S_2, she should take action a, and when she wants S_1', S_2 for some $S_1' \neq S_1$, she should take action $b \neq a$ (such sets, or some of the form S_1, S_2', must exist at any nontrivial action of Alice). Suppose Bob plays the following strategy:

- When Alice takes action a, Bob will give her $\overline{S_1}, \overline{S_2}$,
- but when she plays b, Bob will give her S_1', S_2.

(Intuitively, Bob "threatens" to give Alice *neither* of the sets she likes, *unless* Alice lies about S_1.) When Bob plays this strategy and Alice wants S_1, S_2, she'd rather lie and take action b. Thus, the mechanism is not DSIC.

A counterexample

- Issue: Depending on what the mechanism already knows about Bob, he *may not actually be able to pull off the "threat"*
  - E.g. in this "partial mechanism", every type of Alice has a "dominant action" at her first node
  - Still, this is only the case when we "know a lot about Bob's type", so in a poly CC mechanism, this "can't help much"
  - See the paper for details!



Other result: Separating CC and CC^{EPIC}

- We also consider the gap between CC and CC^{EPIC}
- This is essentially the communication needed to **compute payments**
  - [Fadel, Segal 09]: $\exists f:\ CC^{EPIC}(f) \ge CC(f) + 1$
  - [Babaioff, Blumrosen, Shapira 13]: $\exists f$ with n players : $CC^{EPIC}(f) \ge n \cdot CC(f)$

Theorem:
There exists a function G such that $CC^{EPIC}(G) = \exp(CC(G))$.

- Insight: Functions can be *efficient* and *incentivizable*, yet not *efficiently incentivizable*
- Cf. Dobzinski and Ron (also in STOC 2021)
  - Independently gives a different construction for the same separation
  - Answers other questions regarding price computation