The short side advantage in random matching markets
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Preliminaries and Results

**Goal:** investigate the effect of competition on the average case behaviour of stable matching markets

- **n** men and **n + k** women for **k ≥ 0**
  - Uniformly random, full length preference lists
- **[AKL]** found a startling difference between **k = 0** and **k ≥ 1**
  - We give a new, simpler proof

**Observation 1:** Focus on women’s average rank for their husbands
- Smaller is better
  - Women’s average rank for their husbands

**Observation 2:** In balanced market, WPDA terminates as soon as **n** distinct men are proposed to.

**Observation 3:** Therefore, the number of proposals made in WPDA is essentially a coupon collector random variable.

- If numbers from [n] are repeatedly drawn u.a.r., the number of draws needed for every element of [n] to occur gives the coupon collector random variable
- In a random market, each proposal women make is essentially uniform over the **n** men.
- The only difference between WPDA and a coupon collector is that women never propose to the same man twice. (Formally, the coupon collector statistically dominates the number of proposals made.)
- The expected number of draws the coupon collector needs is **O(n log n)**.

**Conclusions:**
- In WPDA, **O(n log n)** proposals are made on average, so women’s average rank for their husband is **O(log n)**.
- In WPDA, men receive **O(log n)** proposals on average, so their rank for their wife is the minimum rank over these proposals, which is **Ω(n/log n)**.

Sharp Transition From **n × n** to **n + 1 × n**

- When **n + 1** women propose to **n** men, the random process terminates when some specific woman has proposed to every man.
- No woman wants to go unmatched, so they keep proposing to men and pushing each other down their preference lists.

**Shift perspective to man-proposing deferred acceptance (MPDA).**

**Investigate the woman-optimal outcome by finding the best strategic manipulation a woman can achieve.**

**Lemma ([IM])**

A woman **w**∗ has a stable partner of rank better than **i** if and only if **w**∗ remains matched in man-proposing deferred acceptance when she truncates her list after rank **i**.

**Unbalanced Market**

- By the lemma, woman **w**∗’s rank for her partner in woman-optimal outcome is the best (i.e. minimum) rank **i** at which she can truncate her list while still being matched in MPDA.
- Consider running MPDA with **n** men and **n + 1** women. Similar to the balanced market, MPDA terminates as soon as **n** distinct women have been proposed to.
- Now imagine **w**∗ rejects all proposals she receives. Run MPDA until all women other than **w**∗ receive a match.
- The number of proposals again follows a coupon-collector random variable, and we expect **O(n log n)** total proposals.
- So **w**∗ should get **O(log n)** total proposals.
- Thus, the expected best (minimum) rank of a proposal she receives is **Ω(n/log n)**. This is also the minimum rank where she can truncate her list and still receive a match, so in expectation she has no stable partners better than this rank.

**Larger Imbalance**

- Fix constant **λ > 0** and consider **n** men and (**1 + λ**)**n** women.
- Women’s average rank for their husbands is **Ω(n)**.
  - More specifically, **Ω(n/log(1 + 1/λ))**.

**Citations**

